**47330 Energy Storage and Conversion**
**Assignment 3**
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**Task 1:**

The annual heat production cost of the plant is 285.13DKK/MWh. This was calculated by finding the cost of running each component and the revenue generated by the CHP. To find the cost of running the CHP we first need to calculate the GJ of fuel needed. This is done by dividing the GJ heat output of the CHP since it has a 75% heat production efficiency. We find the cost of fuel by multiplying this value by 90 DKK/GJ which is the fixed cost of biomass fuel. We then multiply the heat production by two to get the cost of maintenance. To find the revenue generated by the CHP we can multiply the GJ of fuel by 0.3 since the CHP has a 30% electricity efficiency, divide by 3.6 to get the electricity in terms of MWh, and multiply by the current cost of electricity from the grid. To find the total cost of running the CHP we add the cost of maintenance and the cost of fuel and subtract the revenue generated. **[Refer to the appendix for equation 1]**

To determine the cost of running the heat pump we must first find the amount of energy required to run the heat pump. This is done by dividing the energy out by the COP. We can then convert this value to MWh and multiply it by the cost of electricity from the grid. We add the cost of maintenance which is found by multiplying the GJ of heat produced by 2 DKK/GJ and we have the total cost of the heat pump. **[Refer to the appendix for equation 2]**

To determine the cost of running the peak boiler we again need to calculate the GJ of fuel needed which is the boiler output divided by the efficiency. We then multiply this value by the cost of fuel including tax. We add the maintenance cost which is 1 DKK/GJ heat and we have the cost to run the boiler. **[Refer to the appendix for equation 3]**

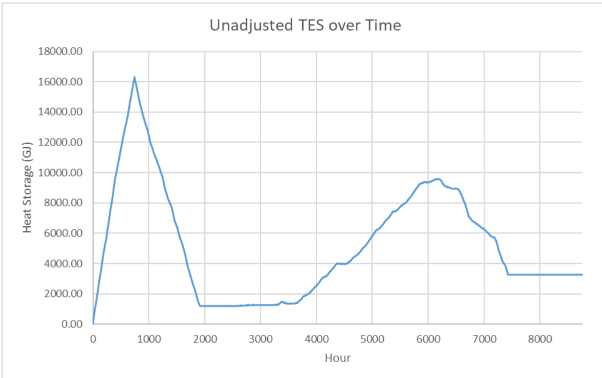
Excel was used to calculate all total costs for each hour and then summed them to find the total heat production cost. To find the cost per MWh we divided the total cost by the heat demand in MWh to get 285.13 DKK/MWh.

All calculations were done in excel using the calculations shown after each section. Variables of the same name are not the same and correspond to the section they follow, except electricity price which is always the price of electricity from the grid. See Excel spreadsheet for calculations with numbers.

**Task 2:**

**Finding Maximum Storage:**

The total waste heat supplied is 28,800 GJ. The thermal energy storage reaches its maximum of 16,330.62 GJ at hour 744. However, at the end of the year, the TES has 3277.69 GJ of energy, which means there is more energy generated than there is demand in the given year. The real maximum energy storage needed is calculated by subtracting the maximum by the minimum storage after the maximum has been reached.



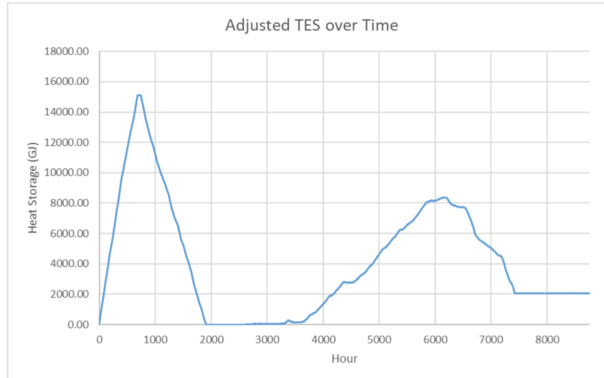




Figure 1: Unadjusted Thermal Storage over Time

Figure 2: Adjusted Thermal Storage Over Time

It can be seen that there is a minimum of 1201.70 GJ after the maximum, which was determined in Excel. This energy is not used, therefore the entire graph after the maximum can be subtracted by the minimum to determine the new, adjusted TES over time. In Figure 2, the actual maximum of the thermal energy storage will be $16330.62 - 1201.7 = 15128.92\ GJ$. The actual waste heat used is calculated as $28800 - 1201.7 = 27598.3\ GJ$. The maximum storage of the TES is found to be 92,647 $m^3$ The conversion from total heat to volume of hot water is given below, the calculation can be found in **Appendix, Equation (4).** $Q = \rho \cdot c \cdot \Delta T \cdot V$ $V = \frac{Q}{\rho \cdot c \cdot \Delta T}$

**Optimal Storage and Cost:**

The average cost per GJ of heat without waste heat is 79.20 DKK, calculated from Task 1. If waste heat is 20 DKK/GJ, each GJ of waste heat used saves on average 59.20 DKK. Since this energy is stored in water, converting energy to volume of water stored shows that each $m^3$ of water stores 0.1633 GJ of energy as can be seen in **Appendix, Equation (5)**

This means that each additional $m^3$ added to storage saves $(0.1633\ GJ)(59.20\ \frac{DKK}{GJ}) = 9.67$ DKK per year on average. This proves that the savings is linearly proportional to the volume of the tank; the largest tank size will produce the most savings, so the optimal size of the TES is the maximum size it can be, which is 92,647 $m^3$.To determine if this investment is economically attractive, first the cost must be calculated. Comparing the two possible TES types available:

Water Pit Storage: $12,300,000\ DKK + (110\frac{DKK}{m^3})(92647\ m^3) = 22,491,170\ DKK$

Storage Tank: $1,500,000\ DKK + (750\ \frac{DKK}{m^3})(92647\ m^3) = 70,985,438\ DKK$

The water pit storage will be used because it is much cheaper to build at the desired volume. The levelized cost of heat is calculated as $LCoH = \frac{itr \cdot I_0 + LC_t}{LE_t}$. We plug in values as seen in **Appendix, Equation (6)**, to get the levelized cost of heat to be 254.36 DKK per MWh. The cost of each heat source was calculated with the same method as in Task 1. The waste heat was calculated as $27598.3\ GJ \cdot 20 =$ 551,966 DKK. The interest rate is 1%, the annual demand is 40,000 MWh, and the lifecycle is 20 years. It's assumed that the leftover 2076 GJ of heat at the end of the year is discharged and each year starts with no heat in the TES.

It is economical to build a TES because the levelized cost is still lower than the 285.14 DKK per MWh cost without TES. This means that with the investment, heat can be sold to consumers at the same price as before and higher profit margins can be achieved.

**Task 3:**

From the given information the slope angle of the PTES is 26.6 degrees and the depth is 16m. Using the known relation, $tan(\theta) = \frac{h}{(a/2-b/2)}$. The formula for the volume of the PTES is $V = (a^2 + b^2 + ab)$ and using both equations we can solve for the top and bottom edge lengths which are a and b respectively. We plug in values and solve for a and b using maple as seen in the **Appendix, Equation (7).** It was found that the top edge of the PTES must be 105.818m and the bottom edge of the PTES must be 41.818m.

**Task 4:**

**Sides and Bottom:**

$Q_{loss} = A_{side} U_{side} (T_{PTES} - T_{soil}), U_{side} = \frac{k_{side}}{t_{side}}$ where k is the thermal conductivity of the soil and t is the thickness of the wall. The same equation is used to calculate the heat loss through the bottom, the only change is the area is now the area of the bottom.

The area of one side is 2640.99m^2, therefore the area of all sides is 10563.97m^2. The wall thickness is 30m, the soil temperature is 10C, the thermal conductivity of the soil is 0.7, and the maximum temperature of the PTES is 80C. Plugging these values into the above equation we find that the maximum heat loss through the sides is 17254.49W. Through the bottom we just plug the area of the bottom into the same equation to find that the heat loss through the bottom is 2857.65W. These calculations can be seen in the **Appendix, Equation (8)**. The total rate of heat loss through the bottom and sides is 20112.14W.

**Top of the PTES:**

The sky temperature is 15 K lower than the ambient air temperature, which is 7.4 C or 280.4 K, so the sky temperature is -7.6 C or 265.4 K. The top of the PTES experiences heat gain through convection and heat loss through radiation. When solving for the temperature of the cover and the heat lost through it we assume that the ambient air temperature is constant at an average of 7.4 C. The thermal conductivity of the cover is 0.045W/m/k, the thickness is 0.25m, and the emissivity is 0.3.

$h_e = h_{convection} + h_{radiation}$

$h_{convection} = 5.8$ W/(m$^2$ K) at a wind speed of 1m/s

$$Q_{radiation} = h_{radiation}(T_c - T_a) = \varepsilon\sigma(T_c + T_{sky})(T_c^2 + T_{sky}^2)(T_c - T_{sky})$$

$$q_1 = \frac{k_{top}}{t_{top}}(T_{PTES} - T_{cover}) \qquad q_2 = h_{convection}(T_c - T_a) + Q_{radiation}$$

Under the steady state assumption: $q_1 = q_2$

$$\frac{k_{top}}{t_{top}}(T_{PTES} - T_c) = h_{convection}(T_c - T_a) + \varepsilon\sigma(T_c + T_{sky})(T_c^2 + T_{sky}^2)(T_c - T_{sky})$$

We can plug the known values into this equation and solve for the temperature of the cover as seen in the **Appendix, Equation (9) and Equation (10).** The temperature of the cover was found to be 279.37K. To find h radiation we use the following equation and the temperature of the cover, which was found to be -18.86 as can be seen in **Appendix, Equation (11)**

$$h_{radiation} = \frac{\varepsilon\sigma(T_c+T_{sky})(T_c^2+T_{sky}^2)(T_c-T_{sky})}{(T_c-T_a)}$$

The total heat loss through the top is found with the following equation.

$$Q_{top} = A_{top} U_{top}(T_{PTES} - T_a), U_{top} = 1/(\frac{t_{top}}{k_{top}} + \frac{1}{h_{radiation}+h_{convection}})$$

Utop was found to be 0.1825 as can be seen in **Appendix, Equation (12)**. The total heat loss is dependent on the ambient temperature and will be calculated in excel. The formula put into EXCEL is **Appendix, Equation (13)** The heat loss through the top is assumed to be dependent only on the ambient temperature but is 0 when the storage is empty. See EXCEL sheet for calculations.

| | |
|---|---|
| Total heat lost b/ | Total heat lost b/s |
| MWh/yr | GJ/yr |
| 48.32 | 173.9409961 |
| Total heat lost to | Total heat lost top |
| MWh/yr | GJ/yr |
| 1221.78 | 4398.414159 |
| Total heat lost | Total heat lost |
| MWh/yr | GJ/yr |
| 1270.10 | 4572.355155 |

**Total annual heat loss:**

The total heat lost by the PTES is 1270.1MWh per year or 4572.35 GJ per year. This was calculated with the following equation. If the energy storage is empty the rate of heat loss through the top is 0 and the heat loss through the bottom and sides is 0 by the scaling factor which means that the total heat loss rate is 0 when the storage is empty which makes sense.

$$\sum_{i=1}^{8760} (Q_{sides,\,i} + Q_{bottom,\,i})(\frac{heat\ stored}{capacity}) + \sum_{i=1}^{8760} Q_{top,i}$$

**PTES heat recovery rate:**

The heat recovery rate can be found using the following equations. It is a rearranged form of the equation given since $annual\ heat\ charged\ =\ annual\ heat\ discharged\ +\ \Delta PTES\ +\ heat\ lost$, which was rearranged to $annual\ heat\ discharged\ +\ \Delta PTES\ =\ annual\ heat\ chraged\ -\ heat\ lost$.

$$heat\ recovery\ rate\ =\frac{annual\ charged\ heat\ -\ heat\ lost}{annual\ charged\ heat}=\frac{annual\ discharged\ heat+\Delta PTES}{annual\ charged\ heat}$$

The recovery rate was found to be 81.61% as can be seen in **Appendix, Equation (15)**. The overall efficiency of the storage is 81.61%.

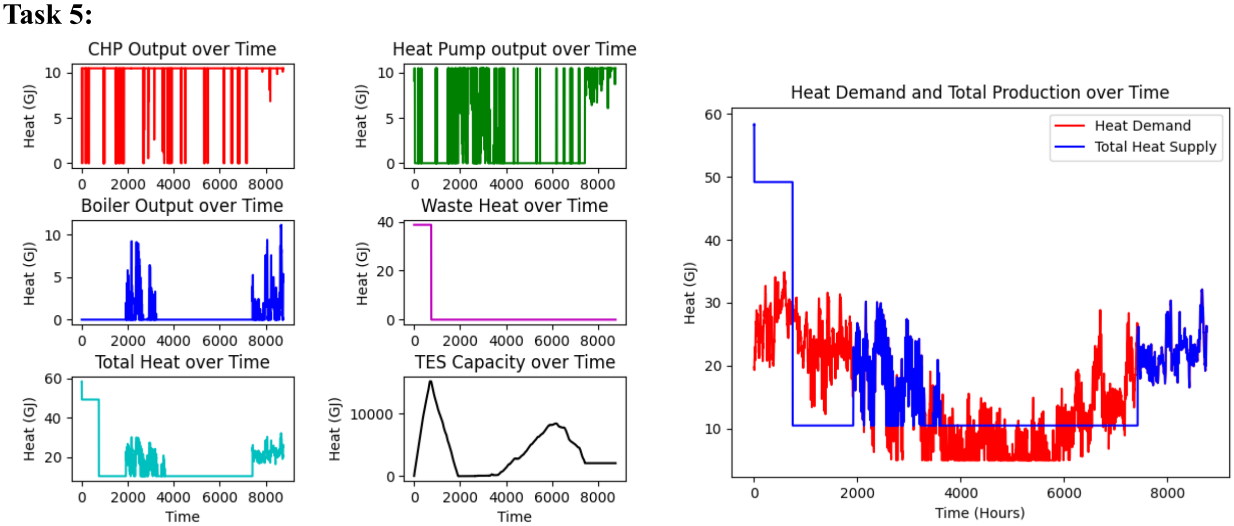
**PTES utilization frequency:**

The utilization frequency can be calculated with the following equation and was found to be 1.64 as can be seen in **Appendix, Equation (16)**.

$$utilization\ frequency\ =\frac{annual\ charged\ heat}{PTES\ capacity}$$

This means that the number of storage cycles per year is 1.64.

**Task 5:**



**Task 6:**

**Analysis of natural gas price fluctuation**

Given the price of 157 DKK/for natural gas in the assignment, a more realistic model, such a sinusoid can be used to simulate the changes in natural gas price over a period of one year. After extensive stipulation, two sinusoids are used to characterize the fluctuation of natural gas prices in one year:

$$A = 6sin((\frac{8x}{365}) - 200) + 180, \qquad B = 20sin(\frac{3x}{365} + 3.3), \qquad C = (A + B) * 0.57$$

Equation A is used to simulate the base cost of natural gas, including the 100 level bias assuming the base price of 100 DKK/GJ as the mean level. A small additional vertical translation is added to account for scaling. The -200 translation allows the function to align more with the natural tendency for natural gas prices to decrease during the summer.

Equation B represents an additional weighting of the gas prices, for a steeper drop during the summer period. Equation B is also horizontally translated for this purpose, as well as weighted greater. The combined sinusoids represent the base cost of natural gas per GJ.

The amplitudes of both sinusoids are arbitrarily chosen for simplicity and ease of visualization.

According to Statistics Denmark (dst.dk, see appendix), the tax added to the price of natural gas per unit is a VAT (value added tax), which equates to a percentage tax. Thus, a flat vertical translation of 57 DKK is not implemented. The unit used on the website is per cubic meter, but we can assume that the VAT tax still applies, even to the price of natural gas per GJ. For simplicity's sake we will stick with the tax rate of 0.57 as given by the assignment.

Equation C combines calculates the VAT tax added to the base cost of natural gas per GJ.

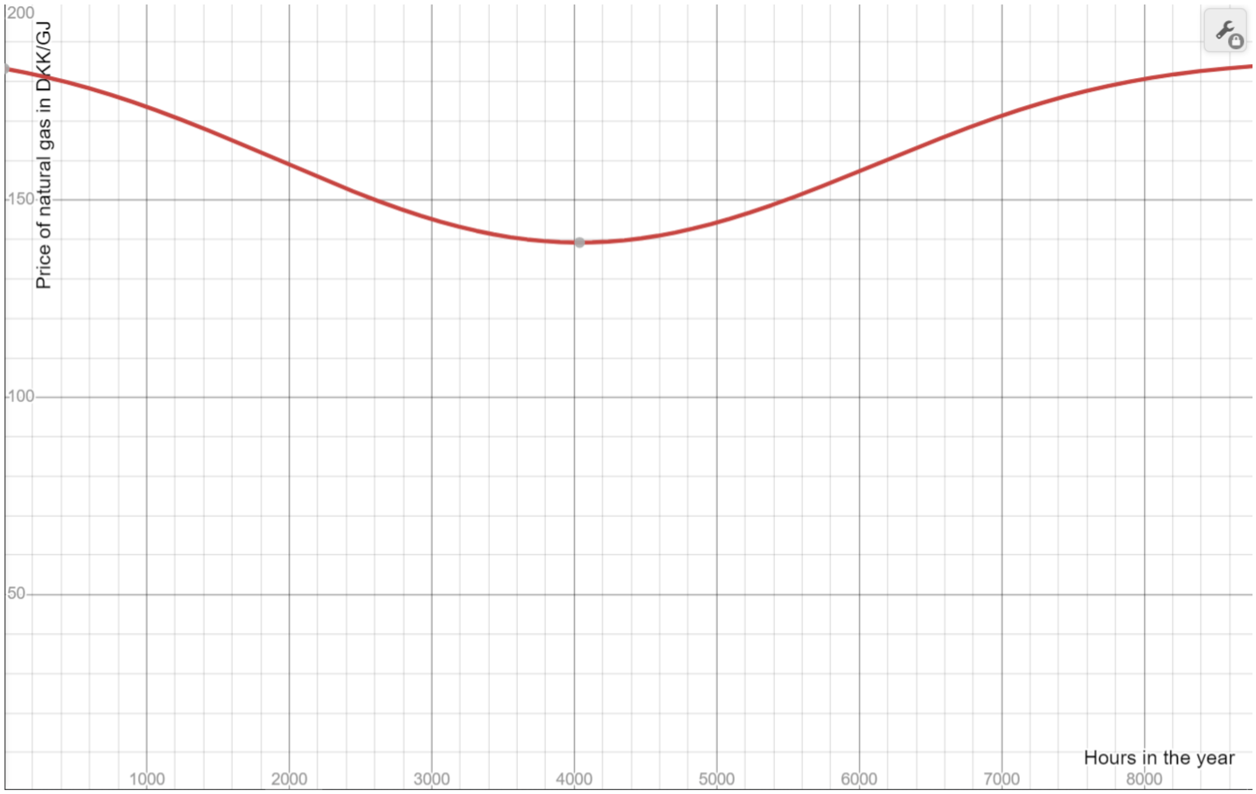


Figure 3: Graph of combined A + B + C equations for the total fluctuating price of natural gas in DKK/GJ (desmos.com)

As expected, there is a simulated drop in natural gas prices around the middle of the year. The mean value of 157 is hit at around the 1st quarter (spring) and the 3rd quarter (fall). The function is biased towards the 0th and the 4th quarters (winter) as expected.

The combined equation is thus input into the calculation for boiler cost. The equations are in terms of days, so the period is extended to be periodic within 8760 hours. To combine the two sinusoids plus tax, the law of distribution is used:

A + B + (A + B)(0.57) = A + B + 0.57A + 0.57B = **1.57A + 1.57B**

The total equation, for the cost of natural gas, per hour, accounting for the tax-weight, in DKK/GJ is:

$\mathbf{Cost_{hour}} = 6 * 1.57 * sin((8x/8760) - 200) + 20 * 1.57 * sin((3x/8760) + 3.3) + 176$

With this equation, an average sinusoidal value of 157 DKK/GJ is obtained.

Using the sinusoidal value per hour inputted into the boiler cost equation, the new boiler cost is 674069 DKK/year.

**Side note:**

According to Statistics Denmark (dst.dk, see appendix), the most current price of natural gas (*for households*) as of half 1 of 2022 is 14.4677 DKK/$m^3$. Given a natural gas density of 0.83 kg/$m^3$, we divide by this amount to result with 17.431 DKK/kg.

According to the World Nuclear Association (world-nuclear.org, see appendix), the heat value of natural gas is 42-55 MJ/kg. We divide by the average value of 48.5 MJ/kg, then scale by 1000 to result with 359.401 DKK/GJ. This finding presents evidence that the original price figure in the assignment may have been incorrect. However, this price is for households, and the price for business may be lower due to bulk purchasing. **See Appendix, Equation (14).**

**Appendix**

**Sources:**
https://www.dst.dk/en/Statistik/emner/miljoe-og-energi/energiforbrug-og-energipriser/el-og-naturgaspriser
https://world-nuclear.org/information-library/facts-and-figures/heat-values-of-various-fuels.aspx

**Equation 1:**

$$fuel = \frac{heat\ output}{0.75},\ fuel\ price = fuel \cdot 90,\ maintenance = heat\ output \cdot 2,$$
$$revenue = \frac{fuel \cdot 0.3}{3.6} \cdot electricity\ price,\ total\ cost = fuel\ price + maintenance - revenue$$

**Equation 2:**

$$energy = \frac{heat\ out}{2 \cdot 3.6},\ energy\ cost = electricity\ price \cdot energy,\ maintenance = heat\ out \cdot 2,$$
$$total\ cost = electricity\ price + maintenance$$

**Equation 3:**

$$fuel\ price = \frac{heat\ out}{0.95} \cdot 157,\ maintenance = heat\ out \cdot 1,$$
$$total\ cost = fuel\ price + maintenance$$

**Equation 4:**

$$V = \frac{(15128.92\ GJ)(1{,}000{,}000\ \frac{kJ}{GJ})}{(972\ \frac{kg}{m^3})(4.2\ \frac{kJ}{kg \cdot K})(353\ K - 313\ K)} = 92,647\ m^3$$

**Equation 5:**

$$1\ m^3 \cdot 972 \frac{kg}{m^3} \cdot 4.2 \frac{kJ}{kg \cdot K} \cdot (353\ K - 313K) \cdot \frac{1\ GJ}{1{,}000{,}000\ kJ} = 0.1633\ GJ$$

**Equation 6:**

$$\frac{(1.01)(22491170)+(20)(9038553)}{(20)(40000)} = 254.36\ DKK$$

**Equation 7:**

$$solve\left(\left\{\frac{2 \cdot 16}{0.5} = a - b,\ \frac{3 \cdot 92647}{16} = a^2 + b^2 + a \cdot b\right\},\ \{a, b\}\right)$$
$$\{a = 105.8180477,\ b = 41.81804770\},\ \{a = -41.81804770,\ b = -105.8180477\}$$

**Equation 8:**

$$Q_{sides} = 4 * 2640.99 * \frac{0.3}{30} * (80 - 10) = 17254.49\ W$$
$$Q_{botton} = 1748.75 * \frac{0.3}{30} * (80 - 10) = 2857.65\ W$$

**Equation 9:**

$$\frac{0.045}{0.25}(353.15 - T_c) = 5.8(T_c - 280.4) + (0.3)(5.6697\times10^{-8})(T_c + 265.4)(T_c^{\ 2} + 265.4^2)(T_c - 265.4)$$

**Equation 10:**

$$solve\left(\frac{0.045}{0.25}\cdot(353 - Tc) = 5.8\cdot(Tc - 280.4) + \left(0.3\cdot5.6697\cdot10^{-8}\right)\cdot(Tc + 265.4)\cdot\left(Tc^2 + 265.4^2\right)\cdot(Tc - 265.4), Tc\right)$$

279.3708764, 252.9931614 + 641.4222853 I, −785.3571991, 252.9931614 − 641.4222853 I

**Equation 11:**

$$h_{radiation} = \frac{(0.3)(5.6697\times10^{-8})(279.38+265.4)(279.38^2+265.4^2)(279.38-265.4)}{(279.38-280.4)} = -18.86$$

**Equation 12:**

$$U_{top} = 1/(\frac{0.25}{0.045} + \frac{1}{5.8-18.86}) = 0.1825$$

**Equation 13:**

$$Q_{top} = 105.818^2 * 0.1825(353 - T_a)$$

**Equation 14:**

$$Price\ of\ natural\ gas,\ DKK/GJ = \frac{14.4677\ DKK/m^3}{0.83\ kg/m^3 * 48.5\ MJ/kg} * 1000$$

**Equation 15:**

$$heat\ recovery\ rate = \frac{26069.66-4572.35}{26069.66} = 0.8161$$

**Equation 16:**

$$utilization\ frequency = \frac{26.69.66}{15128.925} = 1.64$$